Term Paper Problems for Hadron Physics (2017)

2018/1/10 (deadline: 2018/1/31)
Submit your term paper to fuku@nt.phys.s.u-tokyo.ac.jp
(a PDF file typesetted with LaTeX)
no later than 2018/1/31.

Theory I (K. Fukushima)

Problem 1
All well-established hadrons are either mesons ($q\bar{q}$) or baryons (qqq) but there are infinitely more possibilities to form color singlets, say, $qq\bar{q}\bar{q}$ (called tetra-quarks), $qqqqq$ (called penta-quarks), $qqqqqq$ (called di-baryons), etc. It is a big mystery whether such exotic states can exist, and if not, whether there is any QCD-based reason for that. Look up the history, models, relevant experiments, lattice-QCD results, and the current status on tetra-quarks, penta-quarks, and di-baryons (you can pick up not all but some subjects of your choice to study), and summarize what you studied. You can read Wikipedia and internet descriptions, but don't copy and paste them. Make a concise summary in your words.

Problem 2
The most important property for charged particles under a magnetic field B (along the z-axis) is characterized by the following formula of the energy dispersion relation:

$$\epsilon^2 = p_z^2 + 2|eB|(n + 1/2) + m^2 - 2s\, eB\,,$$

where a non-negative integer n refers to the Landau level, m and s represent the mass and the spin of the charged particle. Suppose that all hadron masses are unchanged by the magnetic field, guess what would happen if B can be arbitrarily large. Take $p_z = n = 0$ only and consider if ϵ^2 could be negative for large enough B. Look up $s = 1$ hadron masses and estimate the critical value of B which makes ϵ^2 negative [in the unit of Tesla]. What is theoretically expected once ϵ^2 becomes negative?

Theory II (H. Liang)

Problem 1

For the quantum many-body problems, within a static single-particle picture, the Hartree-Fock-Bogoliubov method employs one of the most general product wave functions of independent quasi-particles. By using this method, the long-range particle-hole correlations and the short-range particle-particle (pairing) correlations can be taken in account on the same footing. Starting from a Hamiltonian with one-body kinetic term and two-body interaction,

$$\hat{H} = \sum_{l_1 l_2} t_{l_1 l_2} \hat{c}^\dagger_{l_1} \hat{c}_{l_2} + \frac{1}{2} \sum_{l_1 l_2 l_3 l_4} v_{l_1 l_2 l_3 l_4} \hat{c}^\dagger_{l_1} \hat{c}^\dagger_{l_2} \hat{c}_{l_4} \hat{c}_{l_3} \,,$$

the Bogoliubov transformation from particles and quasi-particles,

$$\hat{\beta}^\dagger_k = \sum_l \left(U_{lk} \hat{c}^\dagger_l + V_{lk} \hat{c}_l \right) , \qquad \hat{\beta}_k = \sum_l \left(U^*_{lk} \hat{c}_l + V^*_{lk} \hat{c}^\dagger_l \right) ,$$

and the trial wave function of independent quasi-particles,

$$|\mathrm{HFB}\rangle \qquad \text{that} \qquad \hat{\beta}_k |\mathrm{HFB}\rangle = 0 \qquad \text{for all } k \,,$$

please derive the Hartree-Fock-Bogoliubov equation,

$$\sum_{l'} \begin{pmatrix} h_{ll'} & \Delta_{ll'} \\ -\Delta^*_{ll'} & -h^*_{ll'} \end{pmatrix} \begin{pmatrix} U_{l'k} \\ V_{l'k} \end{pmatrix} = E_k \begin{pmatrix} U_{lk} \\ V_{lk} \end{pmatrix} ,$$

where

$$h_{ll'} \equiv (t_{ll'} - \lambda \delta_{ll'}) + \sum_{k'} \sum_{pp'} \bar{v}_{lpl'p'} V^*_{p'k'} V_{pk'} \,,$$

and

$$\Delta_{ll'} \equiv -\frac{1}{2} \sum_{k'} \sum_{pp'} \bar{v}_{ll'pp'} V^*_{p'k'} U_{pk'} \,,$$

with the chemical potential λ and the anti-symmetrized matrix elements $\bar{v}_{1234} = v_{1234} - v_{1243}$. You can refer Chapters 5–7 in *The Nuclear Many-Body Problem* by Peter Ring and Peter Schuck, but please provide clear derivations, in particular, on the coefficient $-1/2$ in the definition of $\Delta_{ll'}$ in the last expression.

Experiment (K. Ozawa)

One of the most important results at RHIC is a measurement of a “jet-suppression”. The suppressions of hadron productions are firstly reported in “Suppression of Hadrons with Large Transverse Momentum in Central Au+Au Collisions at sqrt(s) = 130 GeV”, Physical Review Letters 88:022301, 2002.
Read the paper and answer following questions.

Problem 1
Explain how to measure suppressions of hadrons.
Especially, explain the meaning of the first equation in the paper.

Problem 2
In the paper, π^0 spectra are measured using $\pi^0 \to \gamma\gamma$ decays.
Calculate opening angles of two γs in the Lab Frame for following two cases.
Assume the momentum of the π^0 meson is 4 GeV/c.

1. γs are emitted parallel to the π^0 direction in the π^0 rest frame.
2. γs are emitted perpendicular to the π^0 direction in the π^0 rest frame.

(Convert γ momentum vector from the π^0 rest frame to the Lab frame.)

Problem 3
In the paper, two detectors, such as PbSc and PbGl, are used for the π^0 measurements. Explain the detection principle of these two detectors briefly.