

平成13年度東京大学大学院理学系研究科
物理学専攻修士課程入学試験問題

数 学 ・ 英 語

平成12年8月29日（火） 9時00分～11時30分

【注意事項】

1. 試験開始の合図があるまで、この問題冊子を開いてはならない。
2. 解答には、必ず黒色鉛筆（または黒色シャープペンシル）を使用すること。
3. 問題は全部で5問ある。5問のすべてに解答せよ。
4. 答案用紙は数学3枚、英語2枚（罫線入り）が配布されていることを確かめること。
5. 数学の解答は3枚とじ答案用紙に記入し、1問ごとに別のページを用いること。英語の解答は罫線入りの2枚とじ答案用紙に記入し、同じく1問ごとに別のページを用いること。
6. 各答案用紙の所定欄に**科目名**（数学または英語）、**受験番号**、**氏名**、**問題番号**を記入すること。
7. 答案用紙は点線より切り取られるから、裏面も使用する場合には、点線の上部を使用しないこと。
8. 答案用紙には解答に関係ない文字、記号、符号などを記入してはならない。
9. 解答できない場合でも、答案用紙に科目名・問題番号・受験番号および氏名を記入して提出すること。
10. 答案用紙を草稿用紙に絶対使用しないこと。

数学

[第1問]

4次元ユークリッド空間の部分空間 V_1, V_2 を

$V_1 \equiv \{ \text{条件 } x_1 - x_2 + 2x_3 - x_4 = 0 \quad \text{と} \quad x_1 + 2x_2 - x_3 = 0$
を共に満足するベクトル $\vec{x} = (x_1, x_2, x_3, x_4)$ の集合}

$V_2 \equiv \{ \text{条件 } 2x_1 + x_2 + 2x_3 - 2x_4 = 0 \quad \text{と} \quad 3x_1 + 2x_3 - x_4 = 0$
を共に満足するベクトル $\vec{x} = (x_1, x_2, x_3, x_4)$ の集合}

とする。

- (1) V_1, V_2 はそれぞれ何次元の空間か?
- (2) V_1 と V_2 に共通に含まれるベクトルを求めよ。
- (3) 空間 $V_1 \cap V_2$ は何次元か?

数学

[第2問]

原点を中心とし、周辺を固定した半径 a の薄い円形膜の振動を考えよう。平衡位置からの変位を u とする。極座標を用い $u = u(r, \theta, t)$ とする時、円形膜の振動は波動方程式

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}$$

に従う (簡単のため方程式に現れる係数を 1 とおいた)。以下の設問に答えよ。

(1) n 次のベッセル関数 $J_n(x)$ は微分方程式

$$\frac{d^2 J_n(x)}{dx^2} + \frac{1}{x} \frac{dJ_n(x)}{dx} + \left(1 - \frac{n^2}{x^2}\right) J_n(x) = 0$$

を満たす。 $J_n(x)$ の j 番目の零点を ξ_{nj} とする時、次の直交関係

$$\int_0^1 x J_n(\xi_{nj} x) J_n(\xi_{nl} x) dx = 0, \quad j \neq l$$

が成り立つことを証明せよ。

(2) 境界条件 $u(a, \theta, t) = 0$ と初期条件 $u(r, \theta, 0) = F(r, \theta)$, $\left[\frac{\partial u(r, \theta, t)}{\partial t}\right]_{t=0} = 0$ を満たす波動方程式の解を変数分離の方法で求めよ。(1) で議論した直交関係を用いてよい。

数学

[第3問]

未知関数 $x(t)$ に関する常微分方程式

$$\frac{d^2 x(t)}{dt^2} + 2\alpha \frac{dx(t)}{dt} + \beta^2 x(t) = f(t)$$

について以下の設問に答えよ。ここで、 α および β は正の定数とする。

(1) この方程式をフーリエ変換を用いて調べ、未知関数 x を

$$x(\omega) = G(\omega)f(\omega)$$

の形で表せ。ただし $x(\omega), f(\omega)$ は $x(t), f(t)$ のフーリエ変換とする。

(2) $f(t) = \delta(t)$ (デルタ関数) の場合を考える。(1) の結果を用い留数定理を適用して解 $x(t)$ を以下の場合について求めよ。

(i) $\alpha < \beta$

(ii) $\alpha = \beta$

(iii) $\alpha > \beta$

[第1問] 次の文を読んで、自ら考えるところを英語で、20行程度で記せ。

Nuclear power plants currently provide about 18% of the world's electricity. Although global demand for electricity is increasing, this figure is expected to fall as the construction of new nuclear power stations winds down. Some in the nuclear industry, however, are fighting back, arguing that the only way to meet demand while cutting emissions of greenhouse gases is to build more nuclear power plants.

But this fight back comes at a bad time for the industry. Last year a nuclear worker in Japan died after mishandling enriched uranium. Historically, governments invested heavily in the nuclear industry following the oil crisis in the 1970's. The price of oil then fell, new reserves of oil and gas were discovered. However, much of the optimism within the nuclear industry comes from its environmental credentials. The International Energy Agency estimates that world energy demand in 2020 will be two-thirds more than 1995 levels, and in 1997 at the Kyoto conference industrialized nations pledged to cut greenhouse-gas emissions to 5% below 1990 levels by 2012. Nuclear energy produces almost no greenhouse gases.

Japan remains committed to nuclear power, despite recent problems, and produces about one-third of its electricity from nuclear sources. Japan expects to increase its nuclear capacity by more than 20% over the next two decades. This is not typical of developed countries.

(adapted from *Physics World*, April 2000)

[第2問] 次の英文は、ノーベル平和賞を受賞した或る物理学者の文章である。これを読み以下の設問に答えよ。

(1) 下線部 (a) (b) を和訳せよ。

(2) 科学者になろうとする人に筆者が提唱していることを日本語で(直訳ではなく、自分の言葉で)説明せよ。

The tremendous advances in pure science made during the 20th century have completely changed the relation between science and society. Through its technological application, science has become a dominant element in our lives. It has enormously improved the quality of life. It has also created great perils, threatening the very existence of the human species. Scientists can no longer claim that their work has nothing to do with the welfare of the individual or with state policies.

However, (a) many scientists still cling to an ivory tower mentality founded on precepts such as "science should be done for its own sake", "science is neutral", and "science cannot be blamed for its misapplication." This amoral attitude is in my opinion actually immoral, because it eschews personal responsibility for the likely consequences of one's actions.

The ever-growing interdependence of the world community offers great benefits to individuals, but by the same token it imposes responsibilities on them. Every citizen must be accountable for his or her deeds. This applies particularly to scientists, for the reasons I have outlined. It is also in their interest, because the public holds scientists responsible for any misuse of science. (b) The public has the means to control science by withholding the purse or imposing restrictive regulations. It is far better that scientists themselves take appropriate steps to ensure responsible application of their work.

Professional organization of scientists should work out ethical codes of conduct for their members, including the monitoring of research projects for possible harm to society. It is particularly important to ensure that new entrants into the scientific profession are made aware of their social and moral responsibilities. One way would be to initiate a pledge for scientists, a sort of Hippocratic oath, to be taken at

graduation. As in the medical profession, the main value of such an oath might be symbolic, but I believe it would stimulate young scientists to reflect on the wider consequences of their intended field of work before embarking on a career in academia or industry.

I like the pledge initiated by the Student Pugwash Group in the United States, which has already been signed by thousands of students from many countries. It reads: "I promise to work for a better world, where science and technology are used in socially responsible ways. I will not use my education for any purpose intended to harm human beings or the environment. Throughout my career, I will consider the ethical implication of my work before I take action. While the demands placed upon me may be great, I sign this declaration because I recognize that individual responsibility is the first step on the path to peace."

(precept: guide for behavior, eschew: keep oneself away from)

(adapted from *Science*, 288 (1999))