平成１７年度東京大学大学院理学系研究科
物理学専攻修士課程入学試験問題

英　語　・　数　学

平成１６年８月３１日（火）　９時００分～１１時００分

【注意事項】

1. 試験開始の合図があるまで、この問題冊子を開いてはならない。
2. 解答には、必ず黒色鉛筆（または黒色シャープペンシル）を使用すること。
3. 問題は全部で４問ある。４問のすべてに解答せよ。
4. 答案用紙は数学２枚、英語２枚（罫線入り）が配布されていることを確かめること。
5. 数学の解答は２枚とじ解答用紙に記入し、１問ごとに別のページを用いること。英語の解答は罫線入りの２枚とじ解答用紙に記入し、同じく１問ごとに別のページを用いること。
6. 各答案用紙の所定欄に**科目名**（数学または英語）**、受験番号、氏名、問題番号**を記入すること。
7. 答案用紙は点線より切り取られるので、裏面も使用する場合には、点線の上部を使用しないこと。
8. 答案用紙には解答に関係ない文字、記号、符号などを記入してはならない。
9. 解答できない場合でも、答案用紙に科目名・問題番号・受験番号および氏名を記入して提出すること。
10. 答案用紙を草稿用紙に絶対使用しないこと。

数　学

第１問

(1) n 個の n 次元列ベクトル $\boldsymbol{u}_j (j = 1, \cdots, n)$ を用いて，行列 U を $U = (\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_n)$ で定義するとき，U がユニタリー行列ならば，$\{\boldsymbol{u}_j\}$ は正規直交系をなすことを示せ。

(2) A を実対称行列，$\boldsymbol{x} = (x_1, \cdots, x_n)^T$ を n 次元ベクトルとする。ただし，T は転置を表す。このとき実２次形式 $\phi(\boldsymbol{x}) = \boldsymbol{x}^T A\boldsymbol{x}$ に対し適当な直交変換 $\boldsymbol{x} = P\boldsymbol{y}$ を行うと，対角行列 B を用いて $\phi = \boldsymbol{y}^T B\boldsymbol{y} = \sum_{i=1}^{n} \lambda_i y_i^2$ の標準形に変換できる。
$\phi(\boldsymbol{x}) = 4x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 + 2x_2x_3 - 2x_3x_1$ について A，P，$\lambda_i \quad (i = 1, 2, 3)$ を求め標準形で表わせ。

(3) A を n 次実対称行列として，次の微分方程式を考える。

$$\frac{d\boldsymbol{x}}{dt} = A\boldsymbol{x}$$

このとき，スカラー関数 $\Phi(\boldsymbol{x})$ を用いて $\dfrac{d\boldsymbol{x}}{dt} = -\boldsymbol{\nabla}_x \Phi(\boldsymbol{x})$ と書けることを $\Phi(\boldsymbol{x})$ の具体形とともに示せ。ただし，$\dfrac{d\boldsymbol{x}}{dt} = (\dfrac{dx_1}{dt}, \cdots, \dfrac{dx_n}{dt})^T$，$\boldsymbol{\nabla}_x \Phi = (\dfrac{\partial \Phi}{\partial x_1}, \cdots, \dfrac{\partial \Phi}{\partial x_n})^T$ である。

(4) 設問 (3) で軌道 $\boldsymbol{x}(t)$ に沿った Φ の微分 $d\Phi(\boldsymbol{x}(t))/dt$ は，$\dfrac{d\Phi(\boldsymbol{x}(t))}{dt} \leq 0$ を満たすことを示せ。また，任意の $\boldsymbol{x} (\boldsymbol{x} \neq 0)$ に対し $\Phi > 0$ が成り立つならば，解軌道は最終的に $\boldsymbol{x} = 0$ に漸近することを示せ。

第2問

$f(x,t)$ についての偏微分方程式

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}(xf) + D\frac{\partial^2 f}{\partial x^2}$$

を考える。f および $\partial f/\partial x$ は，$x \to \pm\infty$ に対して十分速やかに 0 に収束する。また，D は正の定数とする。

(1) x についての f の定積分

$$I = \int_{-\infty}^{\infty} f(x,t)dx$$

が保存されること，すなわち $dI/dt = 0$ であることを示せ。

(2) 原点 $x = 0$ を中心とするガウス分布関数

$$g(x,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}e^{-x^2/(2\sigma^2)} \qquad (\sigma \text{ は正の定数})$$

が，はじめの偏微分方程式の定常解（すなわち、$\partial f/\partial t = 0$）であるとき，$\sigma$ はどう表せるか。

(3) 中心 $X(t)$，標準偏差 $\sigma(t)$（X および σ は t の関数）のガウス分布関数

$$f(x,t) = g\Big(x - X(t), \sigma(t)\Big)$$

が，はじめの偏微分方程式の解になるために，$X(t)$ および $\sigma(t)$ の満たすべき常微分方程式を求めよ。

(4) はじめの偏微分方程式を，初期条件 $f(x,0) = \delta(x-1)$ のもとで解き，$t \to \infty$ において設問 (2) で求めた定常解に近づくことを示せ。（デルタ関数 $\delta(x-1)$ は，$g(x-1,\sigma)$ の $\sigma \to 0$ の極限であることを考慮せよ。）

英語

第1問

次の英文はH. A. Betheの講演からの抜粋である。これを読み以下の設問(i), (ii), (iii), (iv)に答えよ。

From time immemorial people must have been curious to know what $\boxed{\text{(a)}}$ the sun shining. The first scientific attempt at an explanation was by Helmholtz about one hundred years ago, and was based on the force most familiar to physicists at the time, gravitation. When a gram of matter falls to the sun's surface it $\boxed{\text{(b)}}$ a potential energy

$$E_{\text{pot}} = -GM/R = -1.91 \times 10^{15} \text{erg/g}, \tag{1}$$

where $M = 1.99 \times 10^{33}$ g is the sun's mass, $R = 6.96 \times 10^{10}$ cm its radius, and $G = 6.67 \times 10^{-8}$ the gravitational constant. A similar energy was set free when the sun was assembled from interstellar gas or dust in the dim past; actually somewhat more, because most of the sun's material is located closer to its center, and therefore has a numerically larger potential energy. One-half of the energy set free is $\boxed{\text{(c)}}$ into kinetic energy according to the well-known virial theorem of mechanics. This will permit us later to estimate the temperature in the sun. The other half of the potential energy is radiated away. We have known that at present the sun radiates

$$\varepsilon = 1.96 \text{ erg/g sec.} \tag{2}$$

Therefore, if gravitation supplies the energy, there is enough energy available to supply the radiation for about 10^{15} sec which is about 30 million years.

This was long enough for nineteenth century physicists, and certainly a great deal longer than man's recorded history. It was not long enough for the biologists of the time. Darwin's theory of evolution had just become popular, and biologists argued with Helmholtz that evolution would require a longer time than 30 million years, and that therefore his energy source for the sun was insufficient. They were right.

(1)At the end of the 19th century, radioactivity was discovered by Becquerel and the two Curie's who received one of the first Nobel prizes for this discovery. Radioactivity permitted a determination of the age of the earth, and more recently, of meteorites which indicate the time at which matter in the solar system solidified. On the basis of such measurements the age of the sun is estimated to be 5 milliards of years, within about 10 %. So gravitation is not sufficient to supply its energy over the ages.

Eddington, in the 1920's, investigated very thoroughly the interior constitution of the sun and other stars, and was much concerned about the sources of stellar energy. His favorite hypothesis was the complete annihilation of matter, changing nuclei and electrons into radiation. The energy which was to be set free by such a process, if it could $\boxed{\text{(d)}}$, is given by (2)the Einstein relation between mass and energy and is

$$c^2 = 9 \times 10^{20} \text{erg/g}. \tag{3}$$

This would be enough to $\boxed{\text{(e)}}$ the sun's radiation for 1500 milliards of years. However nobody has ever observed the complete annihilation of matter. From experiments on earth we know that protons and electrons do not annihilate each other in 10^{30} years. It is hard to believe that the situation would be different at a temperature of some 10 million degrees such as $\boxed{\text{(f)}}$ in the stars, and Eddington appreciated this difficulty quite well.

(3)From the early 1930's it was generally assumed that the stellar energy is produced by nuclear reactions. Already in 1929, Atkinson and Houtermans concluded that at the high temperatures in the interior of a star, the nuclei in the star could penetrate into other nuclei and cause nuclear reactions, releasing energy. In 1933, particle accelerators began to operate in which such nuclear reactions were actually observed. They were found to obey very closely the theory of Gamow, Condon and Gurney, on the penetration of charged particles through potential barriers. In early 1938, Gamow and Teller revised the theory of Atkinson and Houtermans on the rate of ≪ thermonuclear ≫ reactions, i. e. nuclear reactions occurring at high temperature. At the same time, von Weizsäcker speculated on the reactions which actually might take place in the stars.

meteorite : 隕石.
1 erg = 10^{-7} J (cgs 単位).
milliard : 10 億.

(i) 文章中の (a) から (f) までの 6 箇所の四角にあてはまる単語の原型を次の中から選べ。

(ア) transform (イ) supply (ウ) keep (エ) get (オ) prevail (カ) occur

(a)		(b)		(c)		(d)		(e)		(f)	

(ii) 下線部 (1) を和訳せよ。

(iii) 下線部 (2) の the Einstein relation とは何であるか，簡単に英語で説明せよ。

(iv) 下線部 (3) を和訳せよ。

第２問

次の英文はR. A. Millikanの講演からの抜粋である。これを読み以下の設問 (i), (ii), (iii) に答えよ。

The most direct and unambiguous proof of the existence of the electron will probably be generally admitted to be found in an experiment which for convenience I will call the oil-drop experiment. But before discussing the significance of that advance I must ask you to bear with me while I give the experimentalist's answer to the very fundamental but very familiar query: ≪ What is electricity? ≫ His answer is naive, but simple and definite. He admits at once that as to the ultimate nature of electricity he knows nothing.

He begins rather with a few simple and familiar experiments and then sets up some definitions which are only descriptions of the experiments and therefore involve no hypothetical elements at all.

He first notes the fact that a pith ball, after contact with a glass rod that has been rubbed with silk, is found to be endowed with the new and striking property that it tends to move away from the rod with a surprisingly strong and easily measurable force. He describes that fact, and affirms at the same time his ignorance of all save the existence of this force, by inventing a new word and saying that the pith ball has been put into a positively electrified state, or simply has received a charge of positive electricity. He then measures the amount of its charge by the strength of the observed force.

Similarly he finds that the pith ball, after contact with an ebonite rod that has been rubbed with cat's fur is attracted, and he proceeds to describe this experiment by saying that it has now received a charge of negative electricity. Whenever the pith ball is found to have been put, by contact with any body or by any other process, into a condition to behave in either of the foregoing ways, it has, by definition, received a charge of either positive or negative electricity. The whole of our thinking about electrical matters starts with these two simple experiments and these two definitions.

In order now to get the most crucial possible test of the correctness or incorrectness of Franklin's conception of a particle, or an atom, of electricity it was clearly necessary to reduce the charge on the pith ball to the smallest possible amount, to change that charge by the most minute possible steps, and then to see whether the forces acting upon it at a given distance from the glass rod (i.e. in a constant field) had any tendency to increase or decrease by unitary steps.

The success of the experiments first performed in 1909, was wholly due to the design of the apparatus, i.e. to the relation of the parts.

The pith ball itself which was to take on the smallest possible charge had of course to be the smallest spherical body which could be found and yet which would remain of constant mass; for a continuously changing gravitational force would be indistinguishable, in its effect upon the motion of the charged body, from a continuously changing electrical charge.

A non-homogeneous or non-spherical body also could not be tolerated; for the force acting on the pith ball had to be measured by the speed of motion imparted to it by the field, and this force could not be computed from the speed unless the shape was spherical and the density absolutely constant. This is why the body chosen to replace the pith ball was an individual oil-droplet about a thousandth of a millimeter in diameter blown out of an ordinary atomizer and kept in an atmosphere from which convection currents had been completely removed by suitable thermostatic arrangements. The glass rod, the purpose of which was to produce a constant electrical field, was of course replaced by the two metal plates C and D (Fig. 1) of an air condenser, one of the plates (D) being attached to the positive, the other (C) to the negative terminal of a battery, and a switch being added, as shown in the figure, so as to make it possible to throw the field on or off at will.

pith: 木髄. 植物の茎の中心にある柔らかい組織. 軽く、帯電しやすい性質を持つ.
rod: 棒.
spherical: 球形の.
convection currents: 対流.
save: を除いて (= except).
atomizer: 噴霧器.

(i) Millikanは木髄球が受け取った電荷が正であるか，あるいは負であるかを調べるためには，どのような実験を行えば良いと言っているだろうか。それぞれの場合の木髄球の振る舞いも答えよ。

(ii) 油滴の落下実験を成功に導いた油滴の特徴について、本文中に述べられていることを箇条書きにしてすべて挙げよ。

(iii) ここには示していないが、原文では実験装置の概略図（Fig. 1）が描かれている。その一部である２枚の金属板（CとD）を含む回路図を，本文中の説明をもとに描け。